Question

There is a one-meter rubber band, with an ant on one end. Each day the ant travels one centimeter. At the end of the day, the rubber band is expanded by one meter. How long will it take for the ant to reach the other side?

Hint

I recommend reviewing the Harmonic Series to help with this one. Here is a link to the Wikipedia page on the topic: https://en.wikipedia.org/wiki/Harmonic_series_(mathematics)

Answer

$e^{99.422784335}$ =~ 15,092,688,622,113,800,000,000,000,000,000,000,000,000,000

Solution

After walking on day 1, the ant will have covered 1cm/1m = 1/100 of the distance. After the stretch at the end of the day, this ratio will not change, as both the distance ahead of and behind the ant stretch at the same rate.

After day 2, the ant will have covered and additional 1cm/2m = 1/200 of the distance. Again, the stretch doesn't change this ratio.

After day 3, the ant will have covered an additional 1cm/3m = 1/300 of the distance.

Considering this series, after day n, the ratio of the distance the ant will have covered is:

1/100 + 1/200 + 1/300 + 1/400 + … + 1/(100*n). =

1/100 * (1/1 + 1/2 + 1/3 + 1/4 + … 1/n)

We need to find such n where this series sums to 1.

To find this n, we consider the hint I have about the Harmonic Series.   A very good estimate of the sum of the harmonic series up to 1/n is …

1/1 + 1/2 + 1/3 + 1/4 + … 1/n = ln(n) + y,

Where y = Euler's constant =
0.57721566490153286060651209008240243104215933593992...

1 = 1/100 * (1/1 + 1/2 + 1/3 + 1/4 + ... 1/n)

100 = 1/1 + 1/2 + 1/3 + 1/4 + ... 1/n

100 = ln(n) + y

100-y = ln(n)

n = e^{100-y} = $e^{99.422784335}$ =~
15,092,688,622,113,800,000,000,000,000,000,000,000,000,000